Modelling dispersal, survival and trapping in SIT trials

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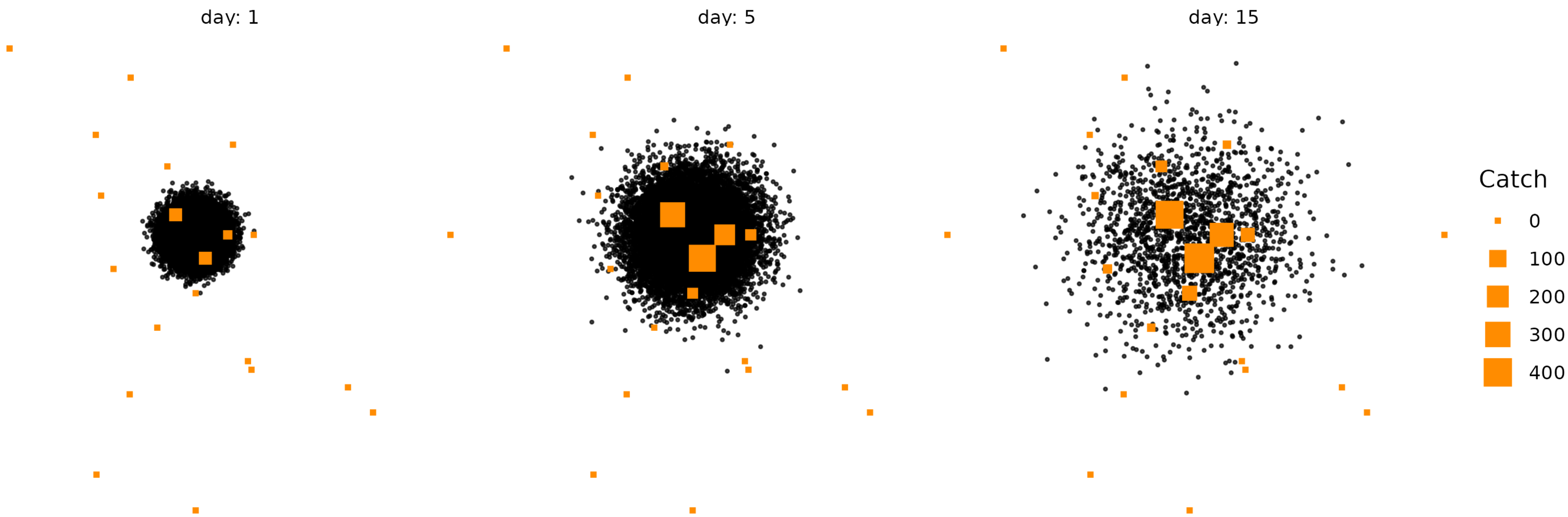
Simulation of the process



Speaker notes

- Thousands of insects (black dots) released from a single point
- They disperse independently from each other following a Brownian motion:
  - Changing direction continuously, with no preferential direction
  - Same model of fluid diffusion at thermal equilibrium
- A few traps (orange squares) scattered over the area catch a fraction of the nearby individuals.
- As time progresses, individuals either eventually get caught or die (naturally, or from other causes such as predators)

Target quantities

Diffusivity

The spread rate of individuals

Survival rate

The fraction of individuals still alive at a given time

Speaker notes

- Diffusivity: the *rate of diffusion*. I.e. The rate at which the particules spread.
- Survival rate: the fraction of individuals still alive at a given time

Observations

Counts of individuals caught during trap activation periods

Example structure

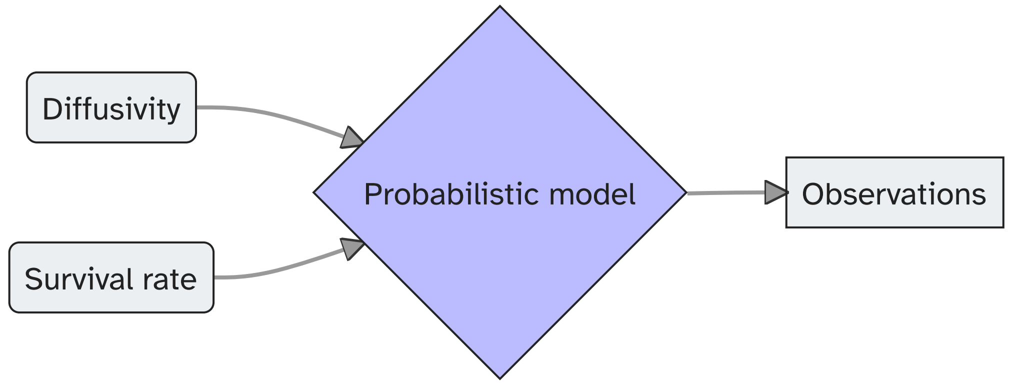
trap	ini_day	end_day	catch
1	0	7	31
2	1	8	15
1	10	15	12
...	...	...	...

(Fake data)

Speaker notes

- Each trap is set up and remains active during a few days
- At the end of the period, operators collect and count their catch

Goal


Diffusivity
Survival rate
Probabilistic model
Observations

Speaker notes

- Develop a probabilistic model for the observations, given the target quantities (forward)
- Use observations to conduct statistical inference on the parameters (backward)

We can only observe dispersal and survival

indirectly

through captures

Speaker notes

Intuitively:

- Captures concentrated in the inner traps for a long time indicate small diffusivity and the converse
- Captures dropping quickly over time indicate low survival rate and the converse

Dispersal

Brownian motion → Diffusion equation:

$\rho(\mathbf{x}, t)$: population density at position $\mathbf{x}$ and time t.

$$\frac{\partial \rho}{\partial t} = D\nabla^2 \rho$$

✅ D: diffusivity coefficient

Speaker notes

- Hypothesis: Brownian motion implies that the dispersion follows the diffussion equation, where
- ∇^2: laplacian operator (divergence of gradient)
- Where we find on of the parameters of interest: the diffusivity coefficient

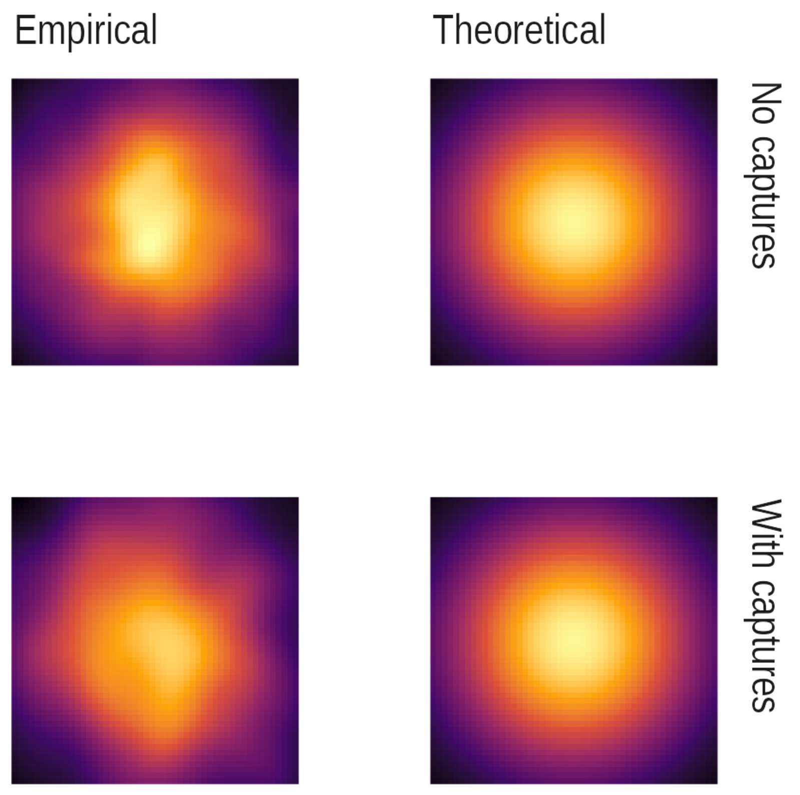
Dispersal: Closed-form solution

$$X_t \mid t, D \sim \mathcal{N}\left(\mathbf{0}, 2tD\mathbf{I}_2\right)$$

Speaker notes

- This simple dispersal model has a very simple analytical expression in terms of a bivariate Gaussian distribution centred at the release point and with dispersion proportional to D and t

Effect of traps on density



Kernel density estimate of the empirical and theoretical distributions at day 15 with or without traps

Speaker notes

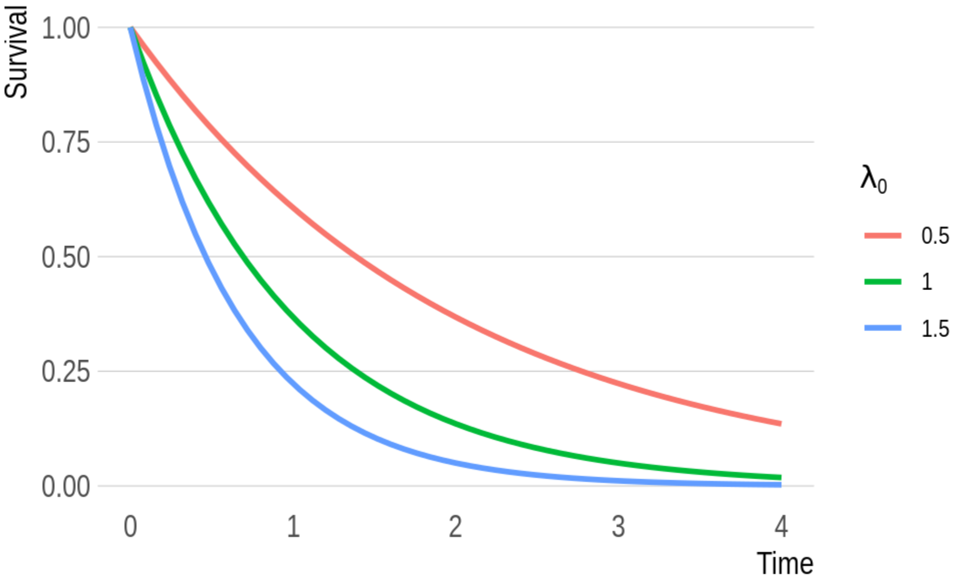
- The capture of insects in traps cause a slight reduction in the density near the origin, with respect to the theoretical density (which ignores catch)
- In practice this is typically irrelevant. Only a problem if there is a significant share of insects caught after many days of capture to make it noticeable (15, in the example)
- However, this is a source of bias in the estimate of the diffusivity coefficient (wich is proportional to the variance of this density)

Survival

Let T_0 be the time of death of individuals.

Assuming a constant hazard rate[1] λ_0,

$$S_0(t) = P(T_0 \leq t)$$
$$= \exp\{-\lambda_0 t\}$$



1. A common, albeit unrealistic simplifying hypothesis.

Speaker notes

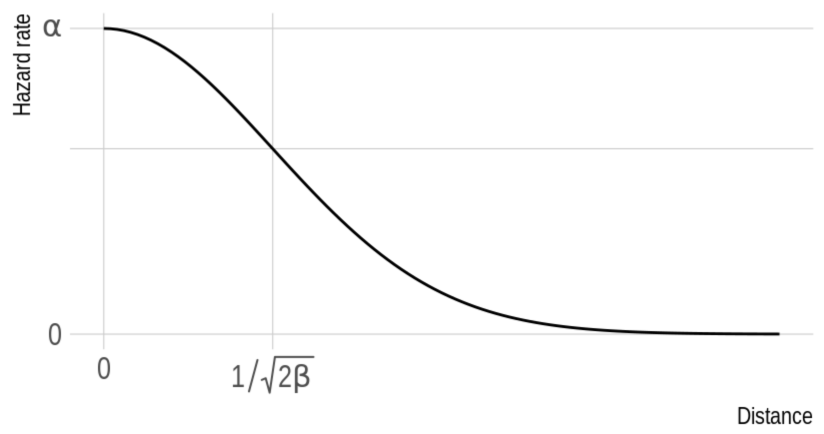
- Hazard rate: instantaneous probability of death, given survival to that time.
- A simple (and common, although maybe not realistic) assumption is that the hazard rate is constant: if an individual survived until t, the probability of death in the next instant is independent of t
- Insects die, eventually, we don't know at which rate
- Furthremore, we can't **observe** their death directly
- When an individual gets caught, we know that they have survived until then

Capture

Assumption: the hazard rate of capture decreases exponentially with the squared distance to the trap

$$h_i(t \mid X_t) = \alpha e^{-\beta||X_t - x_i||^2}$$

$$\alpha, \beta > 0$$



Speaker notes

- We need to model the capture process, even though its parameters are not our target quantities
- For trap i at x_i, conditional to position X_t.

Notation

Random variables associated to a released individual

End Time

$$T > 0$$

Time at which an individual ceases its activity

End Cause

$$C \in \{0, 1, \ldots, I\}$$

Either death ($C = 0$) or capture in trap i ($C = i$)

Speaker notes

- Use concepts and tools from time-to-event (survival) models with competing risks

Specific hazard rates

$$h_i(t) = \mathbb{E}_{X_t}\left[\alpha e^{-\beta||X_t - x_i||^2}\right], \quad \alpha, \beta > 0$$

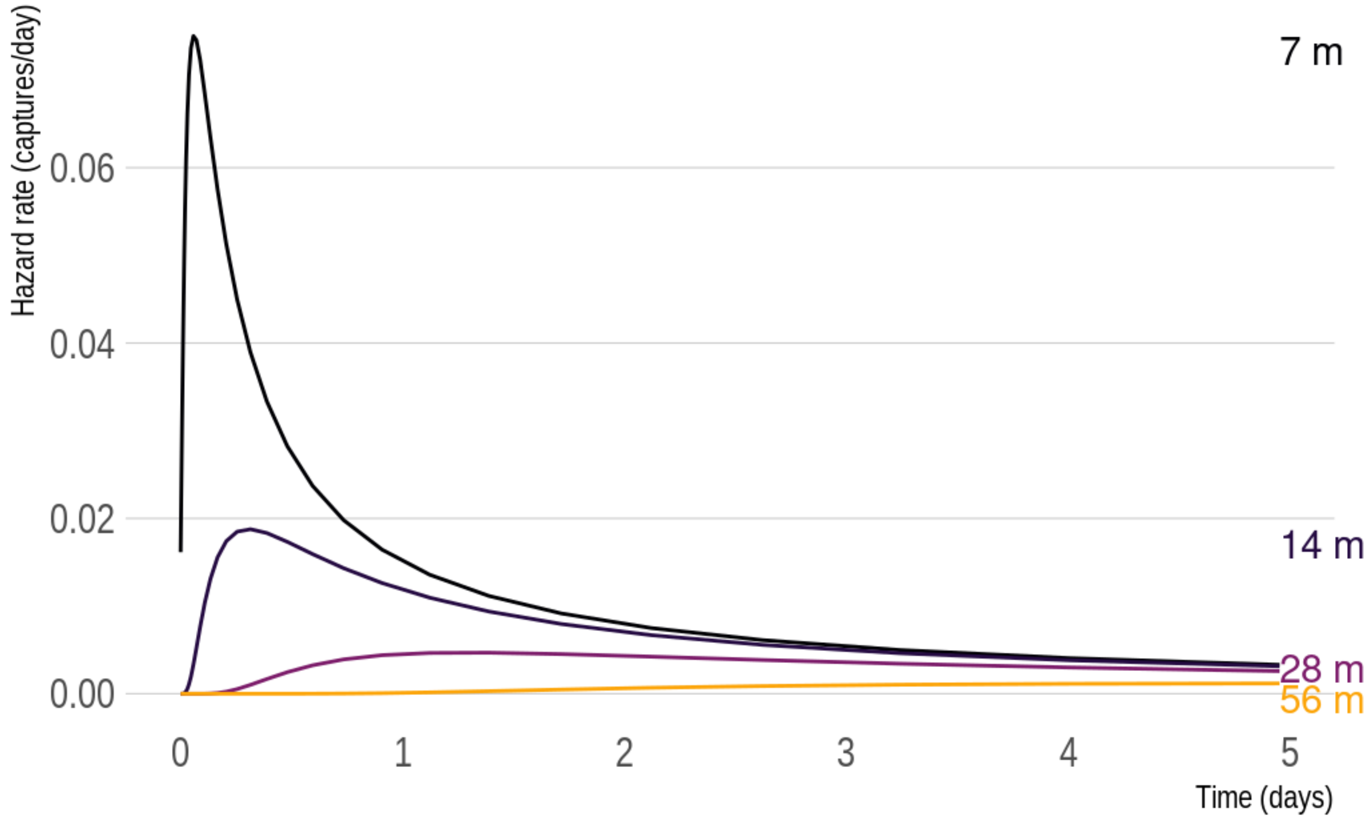
✅ Result

$$h_i(t) = \frac{\alpha}{1 + 4\beta t D} e^{\frac{-\beta||x_i||^2}{1+4\beta t D}}$$

Speaker notes

- We don't know the position of every individual at time t, so we consider the expectation over its (known) probability density
- The calculation yields a closed-form expression for the specific hazard rates

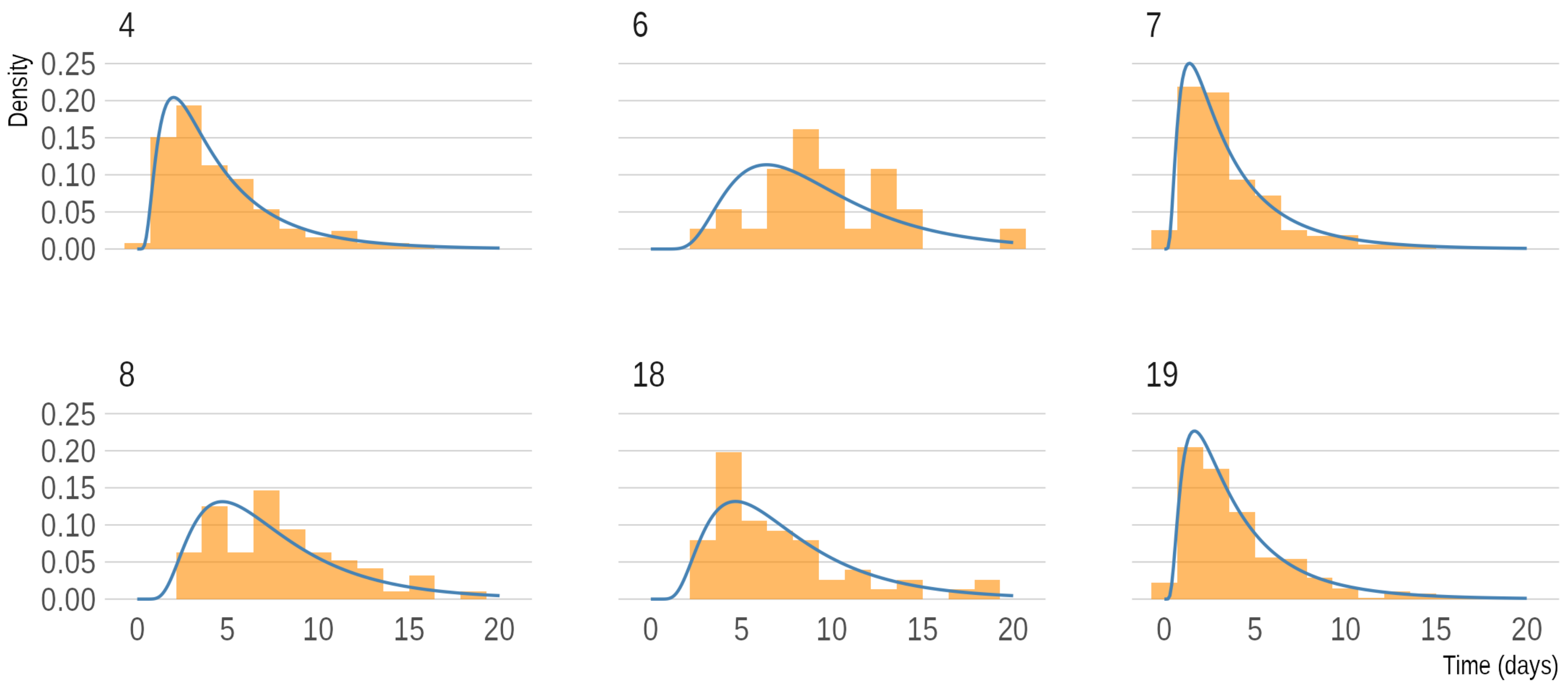
Hazard functions of the capture times for traps at increasing distances from the release location



(Dispersion and capture parameters as in the simulation)

Capture times

Theoretical vs empirical distributions of capture times



(Traps with 10+ total catch)

Mixed joint distribution

At any given time,

1. Additive hazard rates
2. Relative risks of death or capture are proportional to the specific hazard rates

✅ Result

$$\begin{aligned}\mathbb{P}(C = i, T = t) &= \mathbb{P}(C = i \mid T = t) f_T(t) \\ &= h_i(t) S(t)\end{aligned}$$

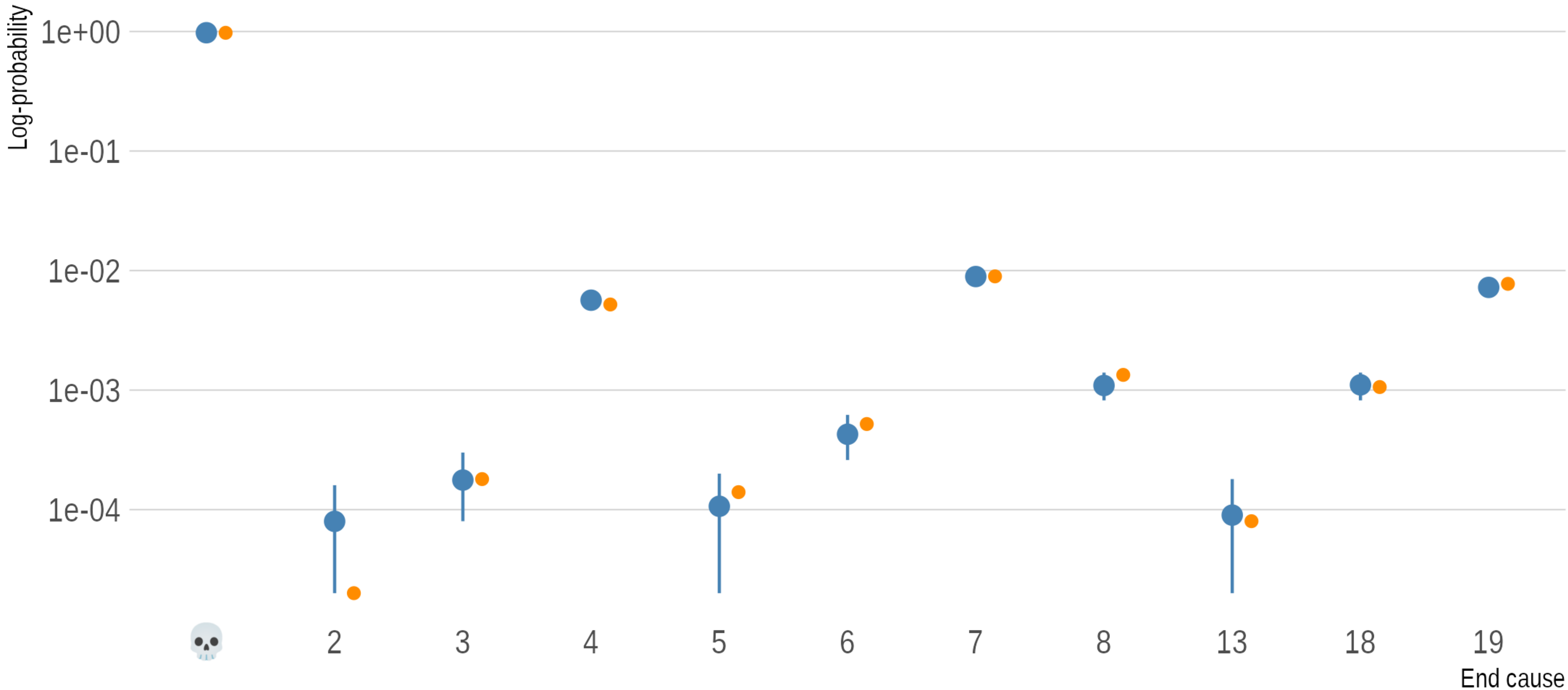
where $S(t) = \exp\left(- \int_0^t \sum_{i=0}^I h_i(u)\, du \right)$

Speaker notes

Borrowing ideas from survival models with competing risks,

- At any given time, the hazard rates are additive
- Enabling the (numerical) computation of the overall survival (i.e. fraction of individuals that are still active at time t)
- ✅ We can integrate the distribution over the observation intervals for each trap to evaluate the likelihood!! 🥳
- 😭 Sadly, we lose the analytical expressions and need to evaluate the integrals numerically
- The exponential component multiplying the specific hazard rate is the overall survival function $S(t)$

Distribution of end causes

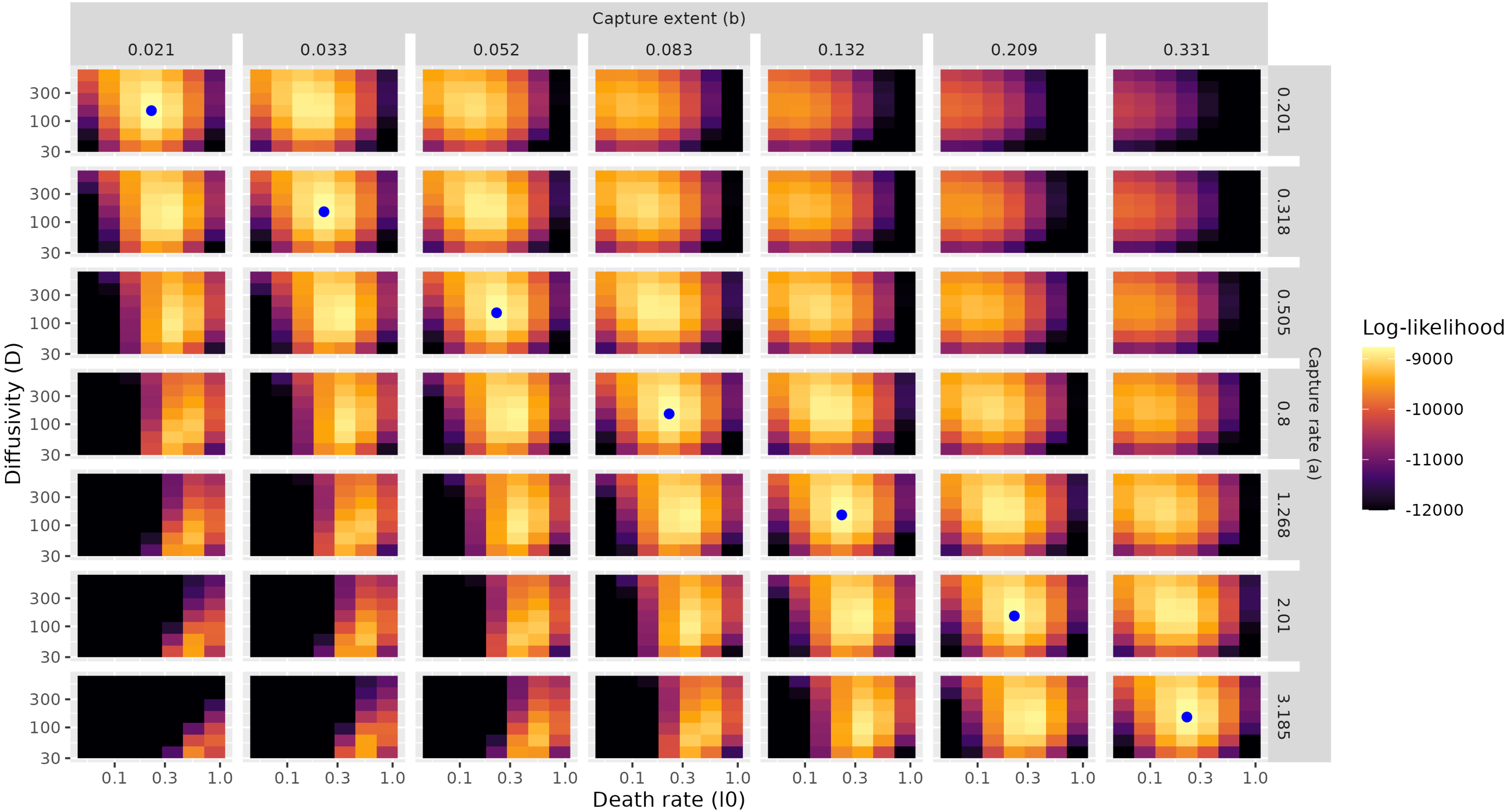


Predicted vs. observed proportion of end events by cause (death / trap)

Speaker notes

- We evaluated the expected and 95% quantile interval for the proportion of dead and caught individuals in each of the traps over all the simulation period and compared with the realised frequency (only traps with 1+ captures shown)

Full likelihood



Speaker notes

- We borrowed ideas from survival models with competing risks to combine the specific hazard rates and compute the joint distribution of end times and causes
- We integrate numerically over the observation periods to obtain the likelihood function
- The figure shows the likelihood function of the simulated scenario.
- The blue points indicate the maximum likelihood (which is almost flat over the diagonal)
- The true value is in the center of the central panel.

SIT: sterile insect technique

Sequence of releases of sterile insects for pest control

- Released males mate with wild females producing sterile eggs

✅ Avoid pesticides (which contaminate the environment)

Introductory video presentation

Speaker notes

- Control disease transmission and reduce agricultural damage without insecticides
- Sterile males mate with wild females reducing offspring
- Need to quantify parameters such as number of individuals to release, temporal frequency, spatial extent, etc. to produce the desired effect of reduction or suppression of the wild population.
- These parameters depend, among other things on the **diffusivity** and the **survival** behaviour of the released sterile males in the context of the specific area.

Issues

- Bias in dispersion (and diffusivity)
- Efficiency in numerical calculations (time consuming)
- Identifiability of capture parameters α and β (reparameterise!)

Extensions

- Non-isotropy
- Varying *attractiveness* of traps
- Integrate further `sit` parameters into the model
- Optimal designs

Speaker notes

- Further parameters such as egg hatching rates, competitiveness…

Conclusions

✅ A formal probabilistic model for the dispersion, survival and capture of insects in `sit` experiments

✅ Enable inference with appropriate quantification of uncertainty in the target parameters

[WIP]

Thank you

- Press 's' to access speaker notes
- Slides available at https://astre.gitlab.cirad.fr/presentations/202408-modah-modelling-sit/
- An extended (23) version of the slides are at https://astre.gitlab.cirad.fr/presentations/202408-modah-modelling-sit/extended.html

Slides made with quarto.

Based on reveal.js and knitr.